A New Positioning Filter: Phase Smoothing in the Position Domain

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ABSTRACT: *Motivated by a requirement to provide real-time meter-level positioning of a NASCAR racing car, a modification of the standard Kalman filter was devised. This paper describes an approach that incorporates previous as well as current position states in a Kalman filter to take advantage of phase measurements differenced over time. In this formulation, the phase measurement difference is a measure of the difference in position in the line-of-sight direction to the satellite, so it can act as a relative position constraint of the current position with respect to the previous one. The formulation of the delta-phase observation equation is described, as well as the modifications made to the Kalman filter to incorporate it. An example used to illustrate the effectiveness of the delta-phase measurements in controlling position error growth is included. Test results in various urban environments are presented.*

INTRODUCTION

During the early days of GPS navigation, a filter was designed that combined a series of delta-phase and pseudorange measurements into a single noise-reduced measurement [1]. While the noise on the measurement used in the navigation solution was reduced, the reduction of the effect of multipath was not as great as had been hoped because of the biased nature of the multipath signal on the pseudorange [2]. At the same time, the time constant in the filter had to be limited because the ionospheric phase advance was a different sign than the pseudorange ionospheric group delay error. Finally, the effectiveness of the phase-smoothing technique was limited because in a kinematic environment, frequent signal outages occur, and every time this happened, all of the smoothed pseudorange information was lost, and the accuracy of the pseudorange reverted back to its nominal unsmoothed level.

Differenced carrier measurements with their associated ambiguities can be used to generate position differences between receivers measuring the same carrier at different locations. Ambiguity estimates are required for this purpose, and deriving these estimates involves time and redundant signals, as well as measurements collected at a base station. There are many examples of this procedure [3, 4]. In an environment in which signals are continuously lost and reacquired, such as in NASCAR racing, ambiguity resolution is often not possible. Furthermore, in a racing environment, the receiver on the cars sometimes tracks fewer than four satellites, making it impossible to generate a position at all without some supplementary means and/or a predictive model. One method for generating the supplementary solution is with the use of a velocity model. Delta-phase measurements can be used to estimate average velocity [5]. This approach helps maintain position accuracy when the constellation drops below four satellites, and also helps reduce the effect of pseudorange errors when the number of satellites is four or more. But the delta-phase measurement measures only average velocity, so some assumptions about the system dynamics must be made. This adds the requirement of additional system noise in the positioning filter, which reduces its accuracy.

This paper describes a method for combining the delta-phase measurement in a filter that includes the current and previous positions (alluded to in [6]). With both the current and previous positions in the filter, a position difference can be derived that is directly observable by the phase difference measured between the previous and current time epochs. The difference between the previous and current positions is completely observable by four phase differences or partially observable if fewer than four satellites are continuously available. This capability is in contrast to and improves upon a position/velocity filter that uses delta phase as

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a velocity estimator, because the delta phase is explicitly treated as a position difference observable, while no assumptions are made about the dynamics of the vehicle. In [7], a delayed state filter is also described that achieves the same effect by eliminating the previous position states from the state vector; it does so by reworking the gains, measurement covariance, and propagation equations to take advantage of the correlation between process noise and measurement noise that results when delta phase is introduced as a position difference observation. The method in which the previous states are maintained was selected because of its simplicity and intuitiveness.

The advantage of this method over phase smoothing is that, for the filter to make use of the delta-phase measurement, it need only be available since the previous time epoch, rather than over the last 50 s or so. Provided that some selection of four satellites is available over every epoch, the position accuracy of the system can be maintained and improved. This is in contrast to the phase-smoothing technique, in which the same four satellites must be continuously tracked for the position accuracy to be maintained and improved by the same amount. In certain environments, various satellites are obstructed periodically. In some cases, the minimum number of satellites may be available for a solution all the time, but it is possible for the tracking duration for all the satellites to be short. In this environment, carrier smoothing the pseudorange is of little help because none of the individual satellites are tracked long enough to reduce the variance for the carrier-smoothed observations. Intuitively, enough information should be available from delta carrier measurements so that the epoch-to-epoch position change can be determined to the level of the delta carrier accuracy, provided at least four delta carrier measurements are available. It is in fact possible to account for all the vehicle dynamics with delta carrier measurements in a least-squares approach [8]. In this method, both the current and previous positions are included as variables in a least-squares adjustment. The idea in this paper is to use the delta carrier measurements as observables in a Kalman filter that incorporates the current position, velocity, and possibly clock, as well as the previous position.

The motivation for this filter approach came from Sportvision, a customer of NovAtel Inc. They wanted to have meter-level positioning accuracy (2 σ) on NASCAR racecars so they could provide real-time computer graphics that followed the cars as they went across the television screen. The navigation difficulty in this problem was that better-than-normal pseudorange positioning was required, but the duration of the satellite constellation was too short for either fixed ambiguity positioning or accurate floating ambiguity positioning. Although the incorporation of track model data into the position solution [9, 10] satisfied (to paraphrase Lincoln) the positioning requirements on some of the tracks all of the time and all of the tracks some of the time, it could not satisfy the positioning requirements on all of the tracks all of the time. The Kalman filter approach, with current position, velocity, and clock states, as well as the previous position state with differential pseudorange and delta carrier measurements as observations, satisfied the requirements to the extent that the technology is used during nearly every race.

During analysis of data collected in the urban canyons of downtown Calgary, it became evident that position errors from a filter that included clock and clock rate estimates would be adversely affected by clock and clock rate errors when the system did not have enough observations to generate an instantaneous position and clock estimate. As a result, the filter was modified so that clock and clock rate parameters were not estimated. Instead, pseudorange, Doppler, and delta-phase measurements were all differenced across satellites before they were used in the Kalman filter to help estimate position and velocity.

In nondifferential mode, the accuracy of the system is at the 1–2 m level when the geometry is good and at the 5–10 m level in urban canyons. In the same urban canyon environment, with a pseudorange-only solution using a least-squares technique, the accuracy often degrades to the 100 m level, so this approach shows a vast improvement over that conventional method. Although no comparisons with a position/velocity Kalman filter are made, it can be said that this method was investigated, but did not give in its unmodified form the results required at the racetrack.

KALMAN FILTER FORMULATION

The Kalman filter is well documented (e.g., [11–13]). It consists of a propagation step and an update step. The Kalman propagation reflects the effects of dynamics over time on the state and of dynamics and time-related uncertainties on the state covariance. The update functions to combine information in the state and its covariance with that of external observations and their covariance, provided some functional relationship exists between the state and the observations. The Kalman filter equations are presented below for reference, along with the specific definitions of the Kalman elements to satisfy position and velocity estimation from GPS observations. The filter element modifications that incorporate the delta-phase measurements into the filter are then described.

Kalman Filter Equations

The specific Kalman filter definition varies with the implementation. The specification of seven basic elements defines the filter to the extent that it can be implemented:

- **x**: state vector
- **P**: state covariance matrix
- **Φ**: transition matrix (differential equation solution)
- **Q**: process noise matrix (effect of incorrect modeling over time)
- **z**: measurement vector
- **R**: measurement covariance matrix
- **H**: linear relationship of measurement to state

Following [11] or [12], the Kalman filter mechanization can be specified as a sequence of state and covariance propagation steps followed by one or more update steps.

Propagation step:

State propagation: $\mathbf{x}_t(-) = \mathbf{\Phi x}_{t-1}(+)$ (1a)

Covariance propagation:

$$\mathbf{P}_t(-) = \mathbf{\Phi P}_{t-1}(+)\mathbf{\Phi}^T + \mathbf{Q} \tag{1b}$$

Update step:

Gain computation: $\mathbf{K} = \mathbf{P}(-)\mathbf{H}^T(\mathbf{HPH}^T + \mathbf{R})^{-1}$ (1c)

State update: $\mathbf{x}(+) = \mathbf{x}(-) + \mathbf{K}(\mathbf{z} - \mathbf{Hx}(-))$ (1d)

Covariance update: $\mathbf{P}(+) = (\mathbf{I} - \mathbf{KH})\,\mathbf{P}(-)$ (1e)

If a position/velocity filter is to be used, the state vector will have six elements. The reference frame used for the computation will be the earth-centered, earth-fixed (ECEF) frame, so the state elements will be

State: $\mathbf{x} = [\delta x, \delta y, \delta z, \delta v_x, \delta v_y, \delta v_z]$

The elements are preceded by the δ symbol to indicate that they are error states, not system elements.

The covariance matrix associated with the pseudorange/delta-phase (PDP) implementation is initialized as a diagonal 6×6 matrix with large diagonal elements. The seed position for the system will be provided by the least-squares process, so the position error states can be assumed to have an initial variance of $(100\ \mathrm{m})^2$, and the velocity error states can be assumed to have an initial variance of $(100\ \mathrm{m/s})^2$.

State initial covariance: **P** = [big diagonal elements, 0 off diagonal elements]

This particular filter maintains only position and velocity states. For the clock components of the system to be eliminated, all pseudorange observations are single differenced (across satellites) to eliminate the common clock offset. All Doppler and delta-phase measurements are also differenced to eliminate clock rate. An example of a covariance matrix for four pseudorange difference observations generated by differencing the first pseudorange with the other four is shown below:

$$\mathbf{R}_d = \begin{bmatrix} \sigma_1^2 + \sigma_2^2 & \sigma_1^2 & \sigma_1^2 & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 + \sigma_3^2 & \sigma_1^2 & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 & \sigma_1^2 + \sigma_4^2 & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 & \sigma_1^2 & \sigma_1^2 + \sigma_5^2 \end{bmatrix} \tag{2}$$

The covariance matrix representing the differenced observation set is nondiagonal to the extent that σ_1^2 is large compared with any of σ_i^2 (for $i \neq 1$). As an implementation note, the phase double differences are not processed as a group but serially, with the correlations between differenced observations being ignored. The effect of the error associated with this simplification is reduced somewhat by choosing a high satellite as the reference in the formation of the differences, because a high satellite will have smaller noise, multipath, and unmodeled atmospheric errors than will a low satellite. Therefore, a covariance with a high satellite as the reference will be closer to diagonal than one with a low satellite as a reference.

The Kalman propagation is dependent on the solution of the differential equations describing the dynamics of the state elements. This propagation contains both deterministic and stochastic portions. Since only position and velocity elements are estimated, the following dynamics matrix describes the state error growth under assumed constant velocity conditions:

$$\delta\dot{\mathbf{x}} = \mathbf{Fx} + \boldsymbol{\omega} \tag{3}$$

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

That is, **F** is a 6×6 dynamics matrix with constant coefficients, and **w** is a vector of white noise forcing functions.

Since the **F** matrix has constant coefficients, the differential equation solution can written as $\mathbf{\Phi}(\Delta t) = e^{\mathbf{F}\Delta t}$. For the **F** matrix in the random walk process seen below, this becomes $\mathbf{\Phi}(\Delta t) = \mathbf{I} + \mathbf{F}\Delta t$, or

$$\mathbf{\Phi} = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

The solution of the deterministic portion provides a transition matrix, and the solution of the stochastic portion provides a $\mathbf{Q}$ matrix. The process noise matrix $\mathbf{Q}$ is based on the transition and the spectral densities $\mathbf{Q}(\tau)$ of the random forcing functions associated with the state according to the equation below (following [14], for example):

$$\mathbf{Q}_{ECEF} = \int_0^{\Delta t} \mathbf{\Phi}(\tau)\mathbf{Q}(\tau)\mathbf{\Phi}(\tau)^T d\tau \tag{6}$$

where $\mathbf{Q}(\tau)$ is a spectral density matrix for the random forcing function vector for the state elements. In general, the spectral densities for the state element forcing functions are not known, and for this filter, they will vary with the system dynamics. So the spectral densities for position and velocity will be chosen heuristically such that the propagated covariance reflects the actual performance of the system. If the theoretical advantage of a local-level spectral density formulation is ignored, the $\mathbf{Q}_{ECEF}$ derivation is simple, and an analytic expression can be generated because the quantity $\mathbf{Q}(\tau)_{VEL\_ECEF}$ is not position dependent. In this case, $\mathbf{Q}(\tau)_{diag}$ is given by

$$\mathbf{Q}(\tau)_{diag} = (q_p, q_p, q_p, q_v, q_v, q_v) \tag{7}$$

with q_v being the common spectral density for all the velocity elements.

Then the $\mathbf{Q}_{ECEF}$ matrix is zero except for the following elements:

$$Q_{11} = Q_{22} = Q_{33} = q_p\Delta t + q_v\Delta t^3/3 \tag{8a}$$

$$Q_{44} = Q_{55} = Q_{66} = q_v\Delta t \tag{8b}$$

$$Q_{14} = Q_{41} = Q_{25} = Q_{52} = Q_{36} = Q_{63} = q_v\Delta t^2/2 \tag{8c}$$

Only the nonzero computed elements are applied to the $\mathbf{P}$ matrix elements. The spectral density for the velocity is derived from the cleaned Doppler misclosures, so the filter is automatically adaptive to changes in system dynamics. Similarly, the spectral densities for position are derived from the delta-phase innovations. That derivation is not central to the description of this filter, so its details are not included.

KALMAN FILTER UPDATE

The linear relationship between the measurements and the state is derived as a matrix of partial derivatives of the functions that link the measurements and the state elements. If such functions do not exist, the state is not observable with the measurement set. Once the linear relationship $\mathbf{H}$ between the state and the measurement set has been determined, the update process follows the update step described earlier.

Finally, pseudorange and Doppler measurements can be used to estimate the state elements. A description of the pertinent linear relationships ($\mathbf{H}$ matrix) follows, first for pseudorange and position, and then for Doppler as it relates to the velocity states.

For the pseudorange difference between satellites i and j and state, a linear relationship can be defined based on the positions of the satellite and the receiver. Assuming the single difference is defined as

$$\Delta\rho^{ij} = \rho^j - \rho^i \tag{9}$$

$$\mathbf{H} = [\Delta x^i/R^i - \Delta x^j/R^j, \Delta y^j/R^i - \Delta y^j/R^j, \Delta z^i/R^i - \Delta z^j/R^j, 0, 0, 0] \tag{10}$$

where $\Delta x^i = x^i - x_r$, the difference between the x components of the i'th satellite and the receiver, with similar expressions for the other difference elements; and $R^i = ((\Delta x^i)^2 + (\Delta y^i)^2 + (\Delta z^i)^2)^{1/2}$ represents the best estimate of the geometric range to the satellite from the receiver.

The measurement that is most closely related to the position in the filter is the reduced pseudorange, that is, the measured pseudorange minus the theoretical pseudorange. Inherent in this process, therefore, is the presumption that a "system" is maintained with the help of the Kalman filter that estimates error states or corrections to the system. In the state update equation using pseudorange differences, $\mathbf{x}(+) = \mathbf{K}(z - \mathbf{Hx}(-))$, $z = \Delta z_m - \Delta z_s$, where Δz_m is the measured pseudorange difference, and Δz_s is the pseudorange difference reconstructed by the system.

For the reduced Doppler difference measurement from satellites i and j, the linear relationship $\mathbf{H}$ with the velocity state is

$$\mathbf{H} = [0, 0, 0, \Delta x^j/R^j - \Delta x^i/R^i, \Delta y^j/R^i - \Delta y^i/R^i, \Delta z^j/R^j - \Delta z^i/R^i] \tag{11}$$

A single reduced Doppler measurement is z_{md} = raw Doppler – satellite clock rate – satellite motion in the line-of-sight direction. The observation used in the Kalman filter is just the difference of two different reduced Doppler measurements. That is, $z = z_{md}{}^j - z_{md}{}^i$. Now a misclosure or innovation, 'w', for the Doppler measurement can be defined as

$$w = z - \mathbf{Hx}(-) \tag{12}$$

MODIFICATION TO INCORPORATE DELTA PHASE

The change in phase measurement over time can provide an estimate of the change in the receiver position over time in the direction of the satellite generating the phase. This measurement would be exact except that over time, changes in satellite position, in tropospheric and ionospheric delay, and in the receiver clock all occur. The measurement is also not normally incorporated in a Kalman filter because the Kalman filter states represent system errors at a particular time, while a delta-phase, or delta position measurement, represents an integrated velocity over time. Thus incorporation of this measurement into the Kalman filter, while attractive, involves some difficulties that must be overcome.

The satellite motion can be accounted for based on the user's knowledge of the satellite orbit. The residual errors in satellite motion resulting from changes in the satellite position error from ephemeris shortcomings are small compared with the atmospheric error changes. The tropospheric and ionospheric error changes are accounted for in part in the error models associated with the measurements and in part by the process noise applied to the position in the propagation portion of the Kalman filter. The clock rate component can be eliminated by differencing delta-phase measurements across satellites (effectively forming double-difference measurements). By using a phase measurement differenced twice across time and satellites, the phase component generated by the change in receiver clock can be eliminated. On this basis, the observation equation relating the phase and delta position is as follows.

The single-difference phase across time can be modeled as

$$\Delta\varphi_{t1t2}{}^{j} = \mathbf{H}^{j}(\mathbf{x}_{t1} - \mathbf{x}_{t0}) + \Delta\text{Clock} \quad (13)$$

where $\mathbf{H}$ is the vector $\mathbf{H}^{j} = [-\Delta x^{j}/R^{j}, -\Delta y^{j}/R^{j}, -\Delta z^{j}/R^{j}]$, and $x_{t1} - x_{t0}$ is the vector of position differences between t_1 (the current time) and t_0 (the previous time). The double-difference phase across time and satellites is

$$\nabla\Delta\varphi_{t1t2}{}^{ij} = \Delta\varphi_{t1t2}{}^{j} - \Delta\varphi_{t1t2}{}^{i} = \nabla\mathbf{H}^{ij}(\mathbf{x}_{t2} - \mathbf{x}_{t1}) \quad (14)$$

where $\nabla\mathbf{H}^{ij}$ is the vector

$$\nabla\mathbf{H}^{ij} = [\Delta x^{i}/R^{i} - \Delta x^{j}/R^{j}, \Delta y^{i}/R^{i} - \Delta y^{j}/R^{j}, \Delta z^{i}/R^{i} - \Delta z^{j}/R^{j}] \quad (15)$$

The only problem with this formulation is that $\nabla\mathbf{H}^{ij}(\mathbf{x}_{t1} - \mathbf{x}_{t0})$, requires that the position at t_1 and the position at t_0 be available. That is, the state must be expanded to include the position at the last epoch.

The state is now defined as

$$\mathbf{x} = [\mathbf{p}_1, \mathbf{v}, \mathbf{p}_0]^{T} \quad (16)$$

where the current position error vector is $\mathbf{p}_1 = [x,y,z]$; the current velocity error vector is $\mathbf{v} = [v_x,v_y,v_z]$; and the previous position error vector is $\mathbf{p}_0 = [x,y,z]$.

The Kalman propagation must be modified not only to support the previously defined dynamics equations for the random walk model, but also to transfer the p_1 elements to the p_0 spot in the state vector during the propagation. That is, the current position after the previous update becomes the previous position after the propagation. At the same time, the current position error is propagated according to the estimated velocity error. The modified transition matrix becomes

$$\mathbf{\Phi} = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (17)$$

Then the update can be applied to an extended state for observation $\nabla\Delta\varphi_{t1t2}{}^{ij}$ with an $\mathbf{H}$ vector

$$\mathbf{H}^{ij} = [\Delta x^{i}/R^{i} - \Delta x^{j}/R^{j}, \Delta y^{i}/R^{i} - \Delta y^{j}/R^{j}, \Delta z^{i}/R^{i} - \Delta z^{j}/R^{j}, 0, 0, 0, -\Delta x^{i}/R^{i} + \Delta x^{j}/R^{j}, -\Delta y^{i}/R^{i} + \Delta y^{j}/R^{j}, -\Delta z^{i}/R^{i} + \Delta z^{j}/R^{j}] \quad (18)$$

applied in the gain computation

$$\mathbf{K} = \mathbf{PH}^{T}(\mathbf{HPH^{T}} + \mathbf{R})^{-1} \quad (19)$$

and the reduced double-difference phase observable is applied to the state via the following update equation:

$$\mathbf{x}(+) = \mathbf{x}(-) + \mathbf{K}[\nabla\Delta\varphi_{t1t2}{}^{ij} - \mathbf{H}^{ij}\,\mathbf{x}(-)] \quad (20)$$

Note that $\mathbf{x}(-)$ and $\mathbf{x}(+)$ are a combination (sum) of state (i.e., system errors) and system.

DELTA-PHASE THEORETICAL EXAMPLE

It is instructive to look at a simplified propagation and update series for a reduced three-state filter representing motion along a single axis. The states consist of the previous and current positions on the axis and the velocity along the axis.

Given the initial state

$$\mathbf{x} = [p_1, v, p_0]^{T} \quad (21)$$

and associated covariance at time t_1

$$\mathbf{P}_0 = \begin{bmatrix} \sigma_{p1}^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_{p0}^2 \end{bmatrix} \quad (22)$$

the simplified transition matrix will be (substituting t for Δt)

$$\mathbf{\Phi} = \begin{bmatrix} 1 & t & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (23)$$

The state propagation gives

$$\mathbf{x}(-) = \mathbf{\Phi}\,\mathbf{x}(+) = \begin{bmatrix} 1 & t & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ v \\ p_0 \end{bmatrix} = \begin{bmatrix} p_1 + tv \\ v \\ p_1 \end{bmatrix} = \begin{bmatrix} p_2 \\ v \\ p_1 \end{bmatrix} \tag{24}$$

and covariance propagation gives

$$\mathbf{P}_t(-) = \mathbf{\Phi} \begin{bmatrix} \sigma_{p1}^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_{p0}^2 \end{bmatrix} \mathbf{\Phi}^T + \mathbf{Q} = \begin{bmatrix} \sigma_{p1}^2 + t^2\sigma_v^2 + q_p t + q_v t^3/3 & t\sigma_v^2 + q_v t^2/2 & \sigma_{p1}^2 \\ t\sigma_v^2 + q_v t^2/2 & \sigma_v^2 + q_v & 0 \\ \sigma_{p1}^2 & 0 & \sigma_{p1}^2 \end{bmatrix} \tag{25}$$

This formulation clearly generates a covariance matrix with highly correlated position elements. In fact, the **P** matrix remains positive definite only because of the uncertainty in the velocity state and the process noise added to the diagonal elements. But a position (or pseudorange) update will affect both the current and to a lesser extent previous position states. Assume the phase measurement geometry is such that all the phase information is in the direction of the modeled axis. Then, the **H** matrix for the phase observation is $\mathbf{H} = [1,0,-1]$. If a single phase observation with a variance of $\sigma_\varphi{}^2$ is used in the update, $\mathbf{R} = \sigma_\varphi{}^2$, and an expression for the gain can be written:

$$\mathbf{K} = \mathbf{PH}^T[\mathbf{HPH}^T + \mathbf{R}]^{-1} = \begin{bmatrix} t^2\sigma_v^2 + q_p t + q_v t^3/3 \\ t\sigma_v^2 + q_v t^2/2 \\ 0 \end{bmatrix} / (t^2\sigma_v^2 + q_p t + q_v t^3/3 + \sigma_j^2) \tag{26}$$

The gain matrix for a small phase variance will be close to 1.0 for the current position element. If there is an error in velocity, say ε_v, then the error in position will be $\varepsilon_p = t\varepsilon_v$, and this error will be reflected in the phase measurement depending on the accuracy of the phase observable and the geometry. In this case the geometry is excellent, so the position error is represented almost entirely by the phase measurement (assume a phase noise increment of η_φ). Therefore, during the phase update, the state correction (assuming for simplicity that the previous state vector was zero) will be

$$\mathbf{x}(+) = \mathbf{x}(-) + \mathbf{K}[-t\varepsilon_v + \eta_j - \mathbf{x}(-)] \cong \mathbf{x}(-) + \begin{bmatrix} -t\varepsilon_v + \eta_j \\ \dfrac{(-t\varepsilon_v + \eta_j)(t\sigma_v^2 + q_v t^2/2)}{t^2\sigma_v^2 + q_p t + q_v t^3/3 + \sigma_j^2} \\ 0 \end{bmatrix} \tag{27}$$

Then, the system's current position will be reduced by almost the exact amount ($t\varepsilon_v$) by which it was in error. So if the geometry is good, and the error on the phase is small, the relative position errors will be nearly eliminated with the phase update. The variance of the position after the update reflects this.

The current position uncertainty during the update is modified according to

$$\mathbf{P}(+) = [\mathbf{I} - \mathbf{KH}]\mathbf{P}(-) \tag{28}$$

The position element of this matrix is

$$\mathbf{P}(+)_{00} = \sigma_{p1}^2 + t^2\sigma_v^2 + q_p t + q_v t^3/3 - (t^2\sigma_v^2 + q_p t + q_v t^3/3)\left(\frac{t^2\sigma_v^2 + q_p t + q_v t^3/3}{t^2\sigma_v^2 + q_p t + q_v t^3/3 + \sigma_j^2}\right) \tag{29}$$

which for a small phase variance reduces to

$$\mathbf{P}(+)_{(0,0)} \cong \sigma_{p1}^2 \tag{30}$$

This is the position variance prior to the Kalman propagation step. The conclusion to be drawn from this example of a simplified system is that the delta-phase measurement can be used with this technique in a Kalman filter to compensate entirely for the degradation in knowledge of position due to velocity error or any other time-related source, provided the phase is accurate enough, and the geometry relating phase change to position change is strong enough.

TEST RESULTS

The results of incorporating the delta-phase measurement can be seen by comparing the plots shown in Figures 1–3. The first set shows some Crescent Heights data, and the second and third show the position improvement through downtown Calgary with its associated urban canyon geography.

Crescent Heights is an older residential neighborhood chosen for its mature tree coverage. The coverage is seen in Figure 1, which shows the number of pseudoranges. The poor coverage later in the run corresponds to the more erratic position results seen at the west side of the trajectory plots that follow.

Now compare the least-squares trajectory with the inertial control trajectory in Figure 2 and the PDP trajectory in Figure 3. The inertial control was generated by NovAtel's inertial system [15], consisting of the integration of an OEM4 receiver operating in differential carrier mode and a Honeywell HG1700-AG11 inertial measurement unit.

The PDP trajectory shows the output of the PDP Kalman filter. The result is a much smoother and more accurate trajectory. The filter is also able to bridge through the portions of the test in which fewer than four satellites are in view. The maximum horizontal position error for this test has been reduced by half—from over 40 m to approximately 20 m. The position availability percentage has increased from 87 to 100 percent (see Tables 1 and 2).

In the urban canyon setting, improvements are more evident. The photograph in Figure 4 and the satellite availability plot in Figure 5 show the tracking environment in the urban core. Not only is the constellation masked, but the receiver tracks a reflected rather than the direct signal on occasion. Figure 5 shows that there are fewer than four satellites available for a significant proportion of the time.

Figure 6 shows least-squares–derived horizontal positions in the downtown corridors. The least-squares trajectory for the first downtown dataset shows very noisy data and clearly demonstrates the effect of unchecked multipath errors. Maximum horizontal position error is approaching 600 m during portions of this dataset.

The PDP trajectory in Figure 7 shows the results of filtering the GPS observations. The solution availability is much improved—to 99 percent (see Table 3). The maximum horizontal position error has been reduced from 600 m to 95 m. The position accuracy in the north/south direction is significantly higher than that in the east/west direction. Since this test is performed primarily driving in east/west directions with high buildings on the north and south of the vehicle, the satellite geometry is such that the along-track direction (east/west) will be better constrained than the across-track (north/south). The satellites in view will be more or less in line with the vehicle's along-track direction, giving relatively good control over the along-track accuracy, but relatively poor control over the across-track accuracy. There is one reset in the trajectory, which can be seen in the far westernmost portion of the southern loop. When the filter propagates long enough with no good updates, it will reset and wait for a good least-squares solution to reinitialize. Although the availability of the least-squares solution is 70 percent in the data shown, the availability in the true urban canyon (southern loop) was only 58 percent. The PDP availability during this highly shaded portion was 98 percent, and the horizontal root-mean-square (RMS) error was 24.7 m (see Table 4).

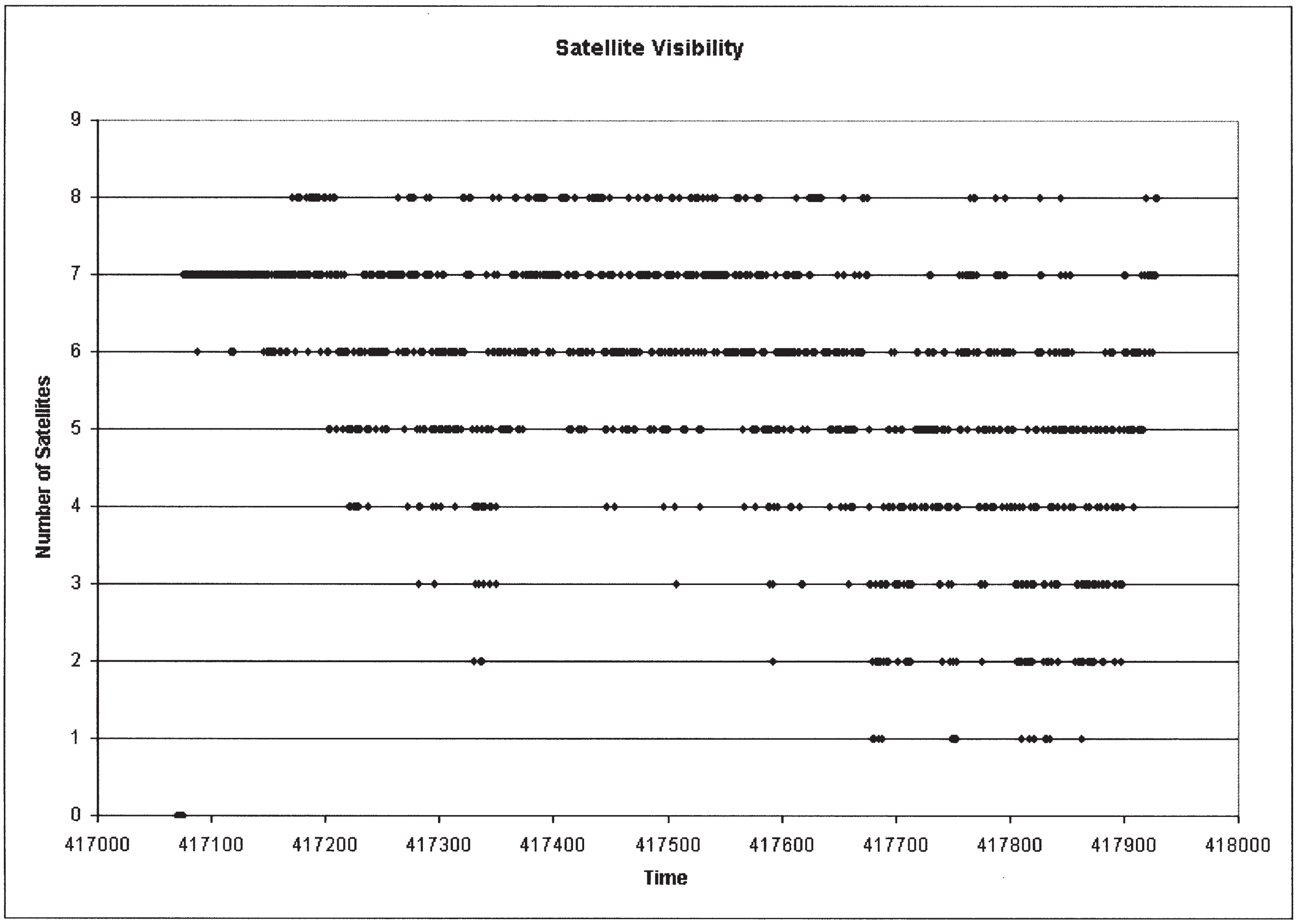

Fig. 1–Crescent Heights Satellite Visibility

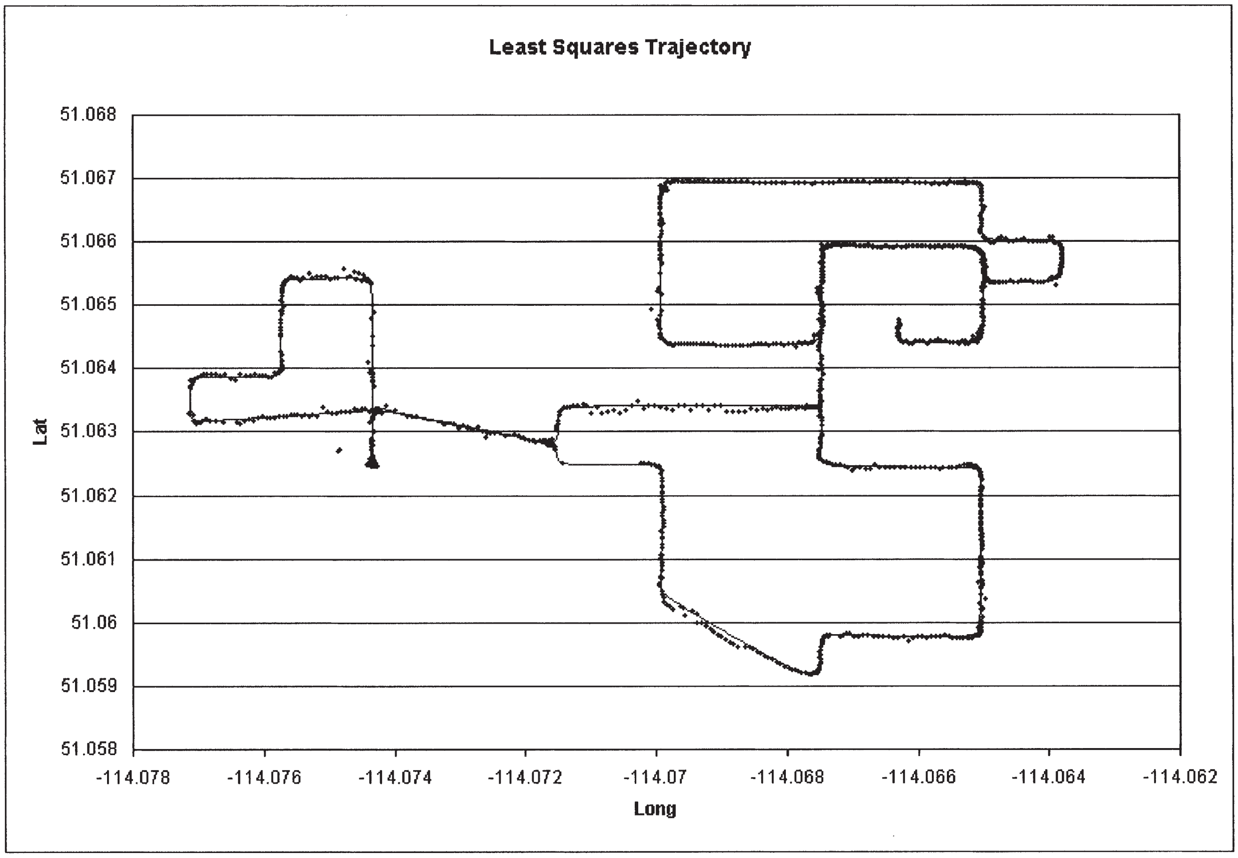

Fig. 2–Crescent Heights Least-Squares Overplot of Inertial Trajectory

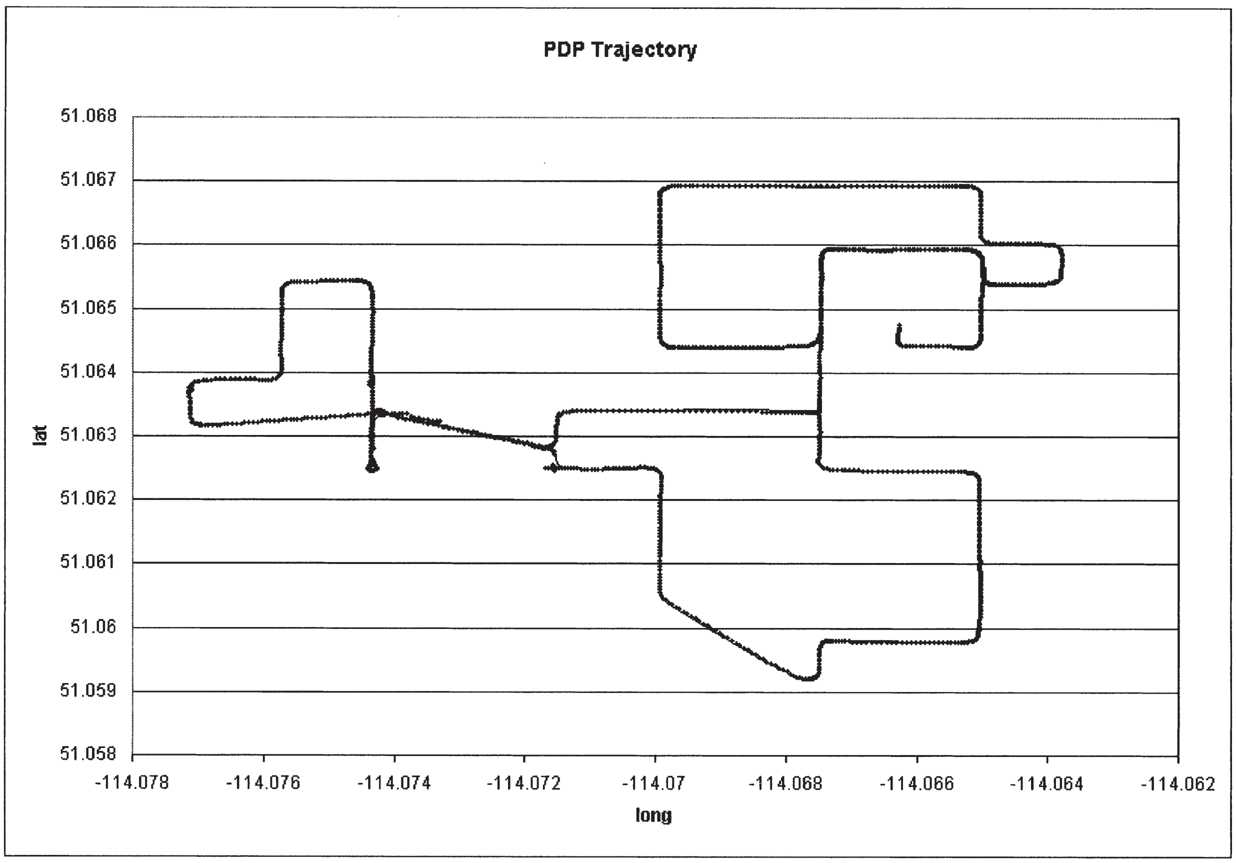

Fig. 3–Crescent Heights PDP Overplot of Inertial Trajectory

Table 1 — Crescent Heights Solution Availability

Parameter	Least Squares	PDP Filter, No Propagated Solutions	PDP Filter, All Solutions
Computed Solution Epochs	1,270	1,351	1,459
Total Possible	1,459	1,459	1,459
% Achieved	87	93	100

Table 2 — Crescent Heights Position Accuracy

Parameter	Least Squares (m)	PDP Filter, No Propagated Solutions (m)	PDP Filter, All Solutions (m)
Latitude Error RMS	3.814	2.799	2.788
Longitude Error RMS	1.784	0.760	0.786
Height Error RMS	13.721	12.509	12.508
2D Position Error RMS	4.210	2.900	2.896

Fig. 4 – Urban Canyon (4th Avenue, Calgary, facing west)

Results for another dataset for downtown Calgary are shown in Tables 5–6 and Figure 8. The line shows inertial control, while the dots show single-point GPS using a least-squares process with only pseudorange inputs. Compare that with Figure 9, which shows the plot of the trajectory of horizontal positions generated with a Kalman filter using pseudorange, Doppler, and delta-phase measurements as inputs.

The PDP trajectory plot in Figure 9 shows the improvement in solution availability. The amount of time a solution is not available is reduced from over 20 percent to only 5 percent. The position spikes from multipath have also been reduced. There are some small deviations from the control solution during periods when few (<4) satellites are available for extended periods of time. There is also one reset of the PDP filter in this data.

CONCLUSIONS

The following conclusions have been generated by this work:

- Delta-phase measurements can be used with this technique in a Kalman filter to compensate for the degradation in knowledge of position due to velocity error or any other time-related source, to the extent that the delta carrier measurements from various satellites are known, and provided the geometry relating phase change to position change is strong enough.

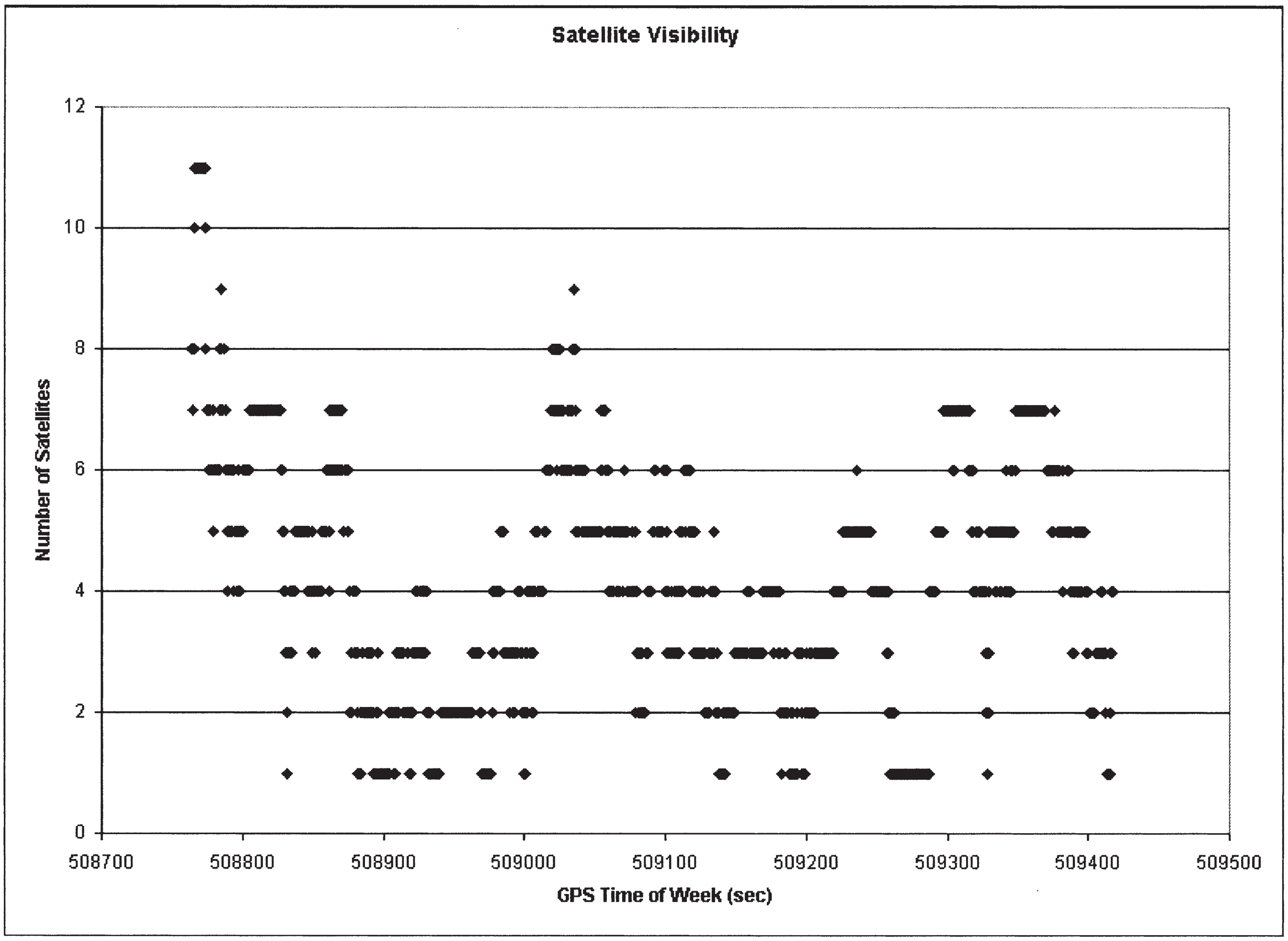

Fig. 5–Urban Canyon Satellite Visibility

Table 3—Urban Canyon Solution Availability

Parameter	Least Squares	PDP Filter, No Propagated Solutions	PDP Filter, All Solutions
Computed Solution Epochs	5,021	6,639	7,103
Total Possible	7,180	7,180	7,180
% Achieved	70	92	99

Table 4—Urban Canyon Position Accuracy

Parameter	Least Squares (m)	PDP Filter, No Propagated Solutions (m)	PDP Filter, All Solutions (m)
Latitude Error RMS	58.359	19.181	19.632
Longitude Error RMS	26.443	4.354	4.454
Height Error RMS	42.038	24.206	26.218
2D Position Error RMS	64.070	19.669	20.130

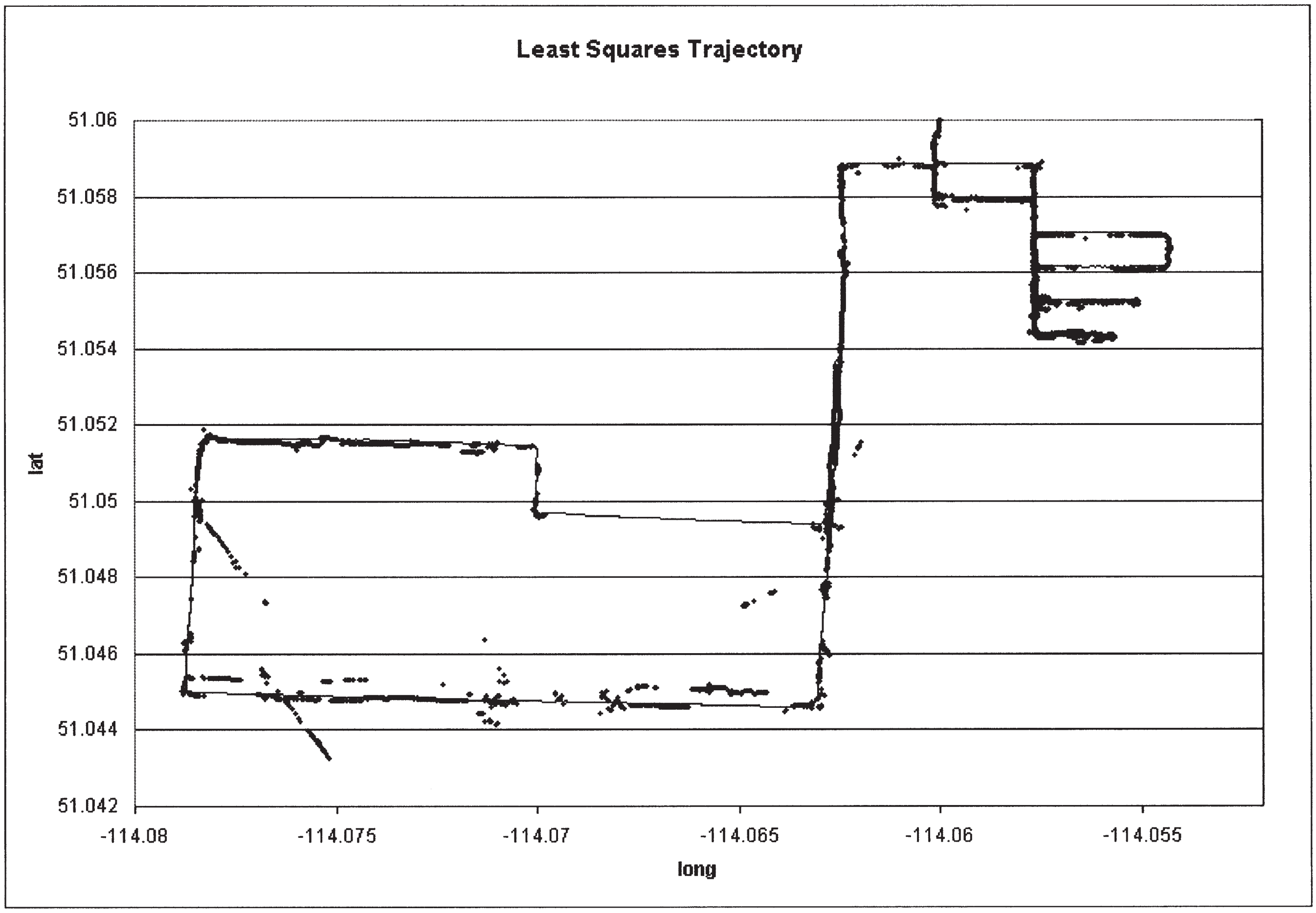

Fig. 6–Urban Canyon Least-Squares Overplot of Inertial Trajectory

Table 5—Urban Canyon Solution Availability

Parameter	Least Squares	PDP Filter, No Propagated Solutions	PDP Filter, All Solutions
Computed Solution Epochs	12,280	14,412	14,749
Total Possible	15,500	15,500	15,500
% Achieved	79	93	95

Table 6—Urban Canyon Position Accuracy

Parameter	Least Squares (m)	PDP Filter, No Propagated Solutions (m)	PDP Filter, All Solutions (m)
Latitude Error RMS	5.988	5.017	5.457
Longitude Error RMS	4.829	2.732	2.786
Height Error RMS	10.737	6.303	6.490
2D Position Error RMS	7.693	5.713	6.127

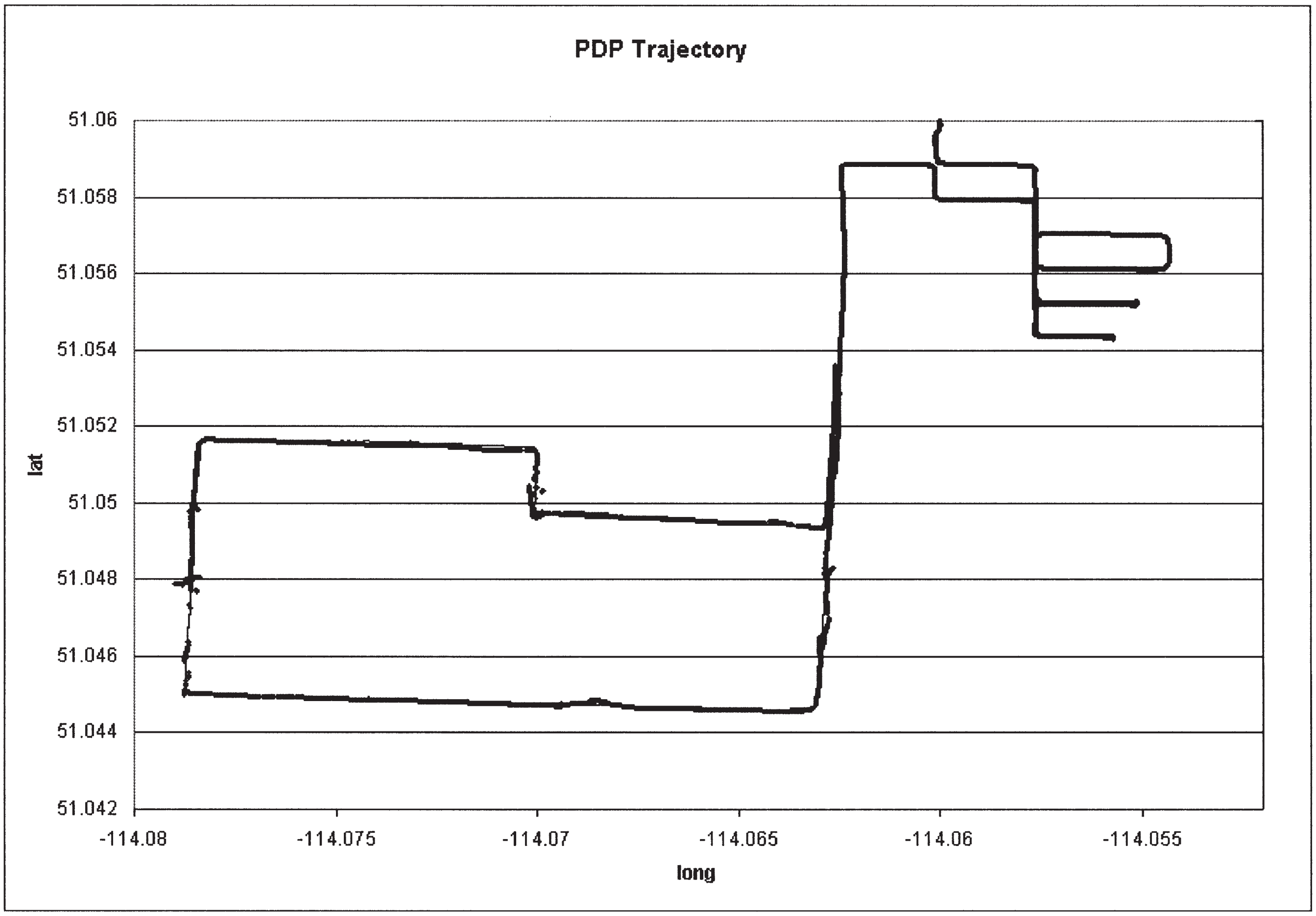

Fig. 7 – Urban Canyon PDP Overplot of Inertial Trajectory

- The advantage of phase smoothing in the positioning domain over phase smoothing in the range domain is that phase-smoothed pseudoranges require continuous tracking of a single observation to contribute effectively to the solution. In the implementation described here, the various statellites can lose lock and be reacquired without significant loss in performance provided at least four satellites (they need not be the same ones) are maintained across the delta time between epochs.
- This method has been shown to improve positioning availability in established residential neighborhoods by over 10 percent and in urban canyon settings by 40 percent.
- This method has improved single-point horizontal accuracy from 4 m (2 dRMS) to 3 m (2 dRMS) in residential neighborhoods. In urban canyon settings, accuracy has improved significantly, from 64 m (2 dRMS) to 20 m (2 dRMS) in one test and from 7.6 m (2 dRMS) to 6.0 m (2 dRMS) in another.
- The single-differenced pseudoranges have significant correlation with one another as a result of the common errors on all observations arising from the reference satellite common to all. Performing the update as a single batch update with a fully populated pseudorange covariance matrix eliminates this issue.
- The correlation also exists for the phase measurements. Its effect is limited by using the highest satellite as a reference (see the discussion of equation (2) above), but investigations should be conducted to determine whether the performance could be improved by processing the delta-phase observations in a batch manner.

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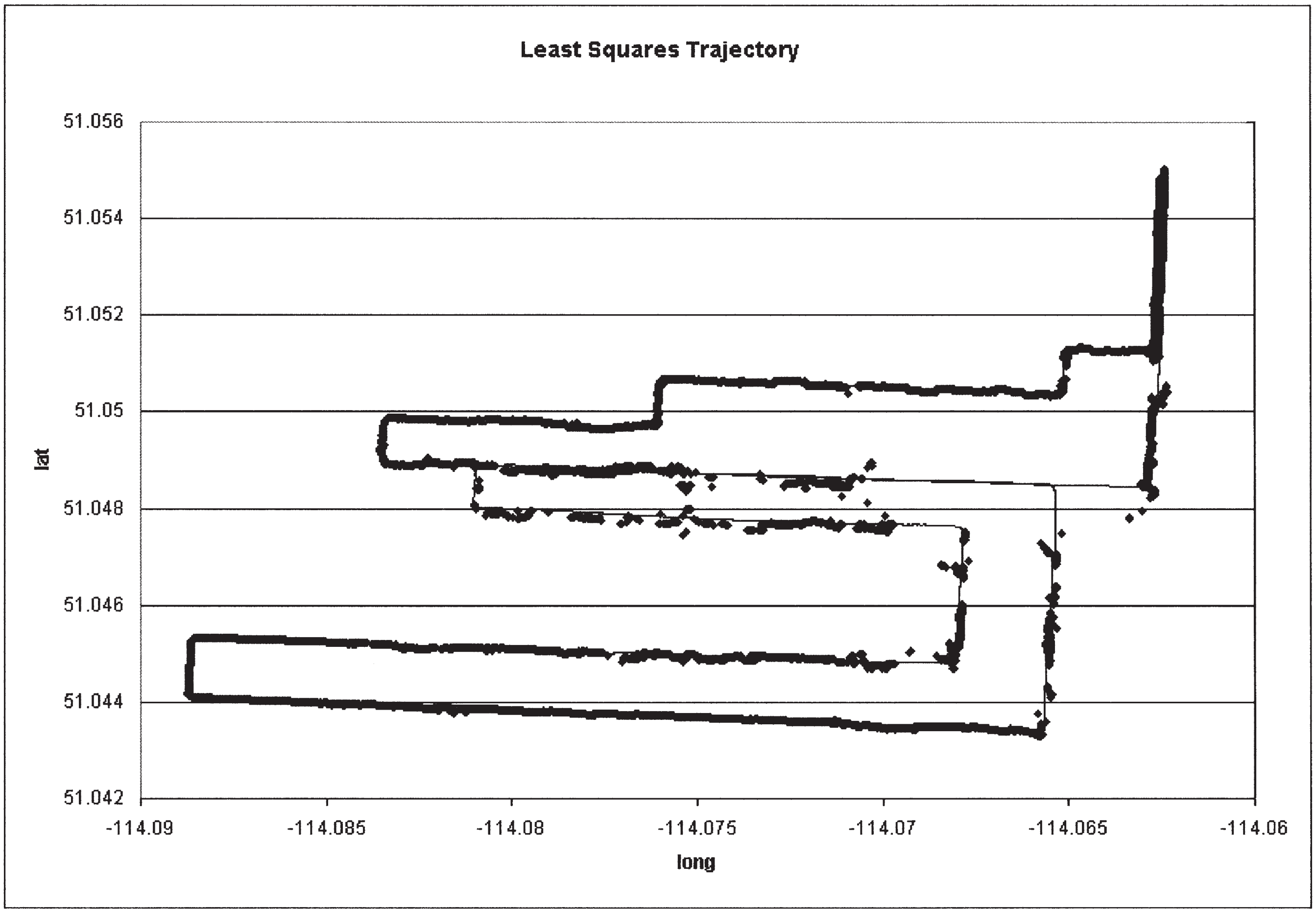

Fig. 8 – Urban Canyon Least-Squares Overplot of Inertial Trajectory

FINAL NOTE

The initial development took place because Sportvision brought us a set of racing environment requirements. The happy ending to that story is that the technology has been deployed successfully by Sportvision, and the results can be seen during televised NASCAR races on either FOX or NBC. Figure 10 shows a sample of the video image from FOX.

Based on a paper presented at The Institute of Navigation's ION GPS-2002, Portland, Oregon, September 2002.

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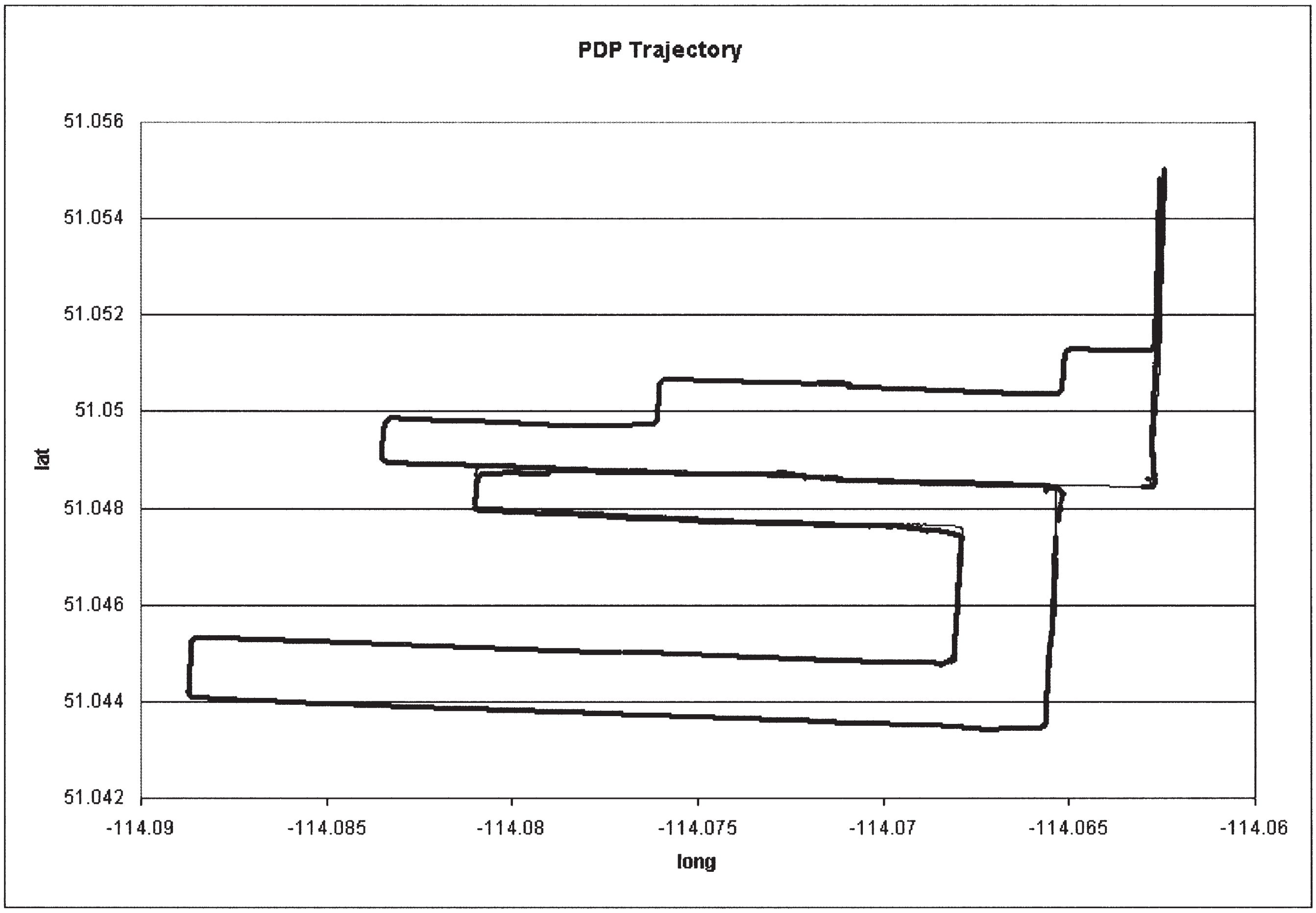

Fig. 9–Urban Canyon PDP Overplot of Inertial Trajectory

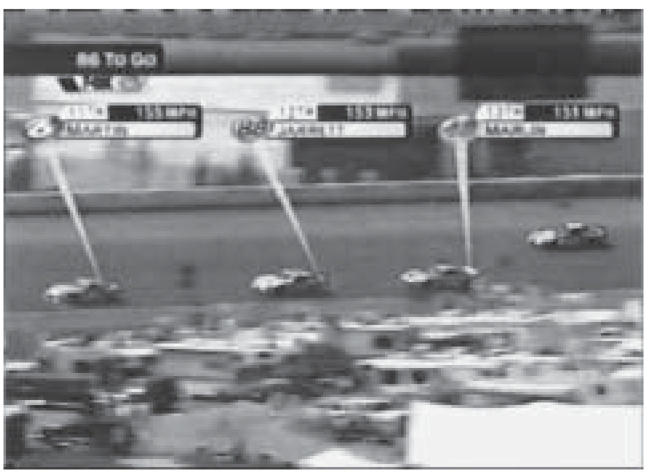

Fig. 10–NASCAR Real-Time Racing Annotation (courtesy of FOX Broadcasting)

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